

The influence of the lengths of turbine blades on the power produced by miniature wind turbines that operate in non-uniform flow fields

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ABSTRACT: Non-uniform flow from a box fan was used to test the power produced by four miniature wind turbines equipped with rectangular blades of different lengths. Flow from the fan was modelled using the solution proposed by Schlichting for a laminar free jet in the plane. Derived analytical results suggested that power produced by turbines immersed in such a flow would increase with the radius of the turbine blades up to a maximum value and, then, decrease with further increases in the size of the radius. Experimental and analytical results were in excellent qualitative agreement: turbines with shorter blades produced more power than those with longer blades consistently, which is the reverse of what might be expected when turbines are subjected to uniform wind speeds. Results indicate that the zone in which power output increases with increasing radii necessitated speeds that were not reached in the tests that were conducted.

INTRODUCTION

According to one-dimensional analysis, the power P, produced by a wind turbine that has a blade radius R and is immersed in a wind stream of uniform speed V is given by [1-6]:

$$P = 2\pi \cdot R^2 \cdot \rho \cdot V^3 \cdot a(1-a)^2, \tag{1}$$

where ρ is the mass density of the air and a is the so-called interference factor [1-6].

Consider two turbines, T_1 and T_2, that are geometrically similar except that R_1, the radius of the blades of turbine T_1 is longer than R_2, that of the blades of turbine T_2. Equation (1) indicates that, when they are immersed in the same uniform wind stream of air, turbine T_1 will generate more power than turbine T_2. Applying Equation (1) twice, the powers produced by turbines T_1 and T_2 are given, respectively, by:

$$P_1 = 2\pi \cdot {R_1}^2 \cdot \rho \cdot V^3 \cdot a_1(1-a_1)^2 \tag{2}$$

and

$$P_2 = 2\pi \cdot {R_2}^2 \cdot \rho \cdot V^3 \cdot a_2(1-a_2)^2. \tag{3}$$

Taking the ratio of the powers produced by these two turbines, one gets:

$$\frac{P_1}{P_2} = \frac{a_1(1-a_1)^2}{a_2(1-a_2)^2}\left(\frac{R_1}{R_2}\right)^2\left(\frac{V_1}{V_2}\right)^3. \tag{4}$$

Equation (4) indicates that the power ratio, $\frac{P_1}{P_2}$, should vary as the square of the ratio of the radii of the turbine blades, if the wind speeds and the interference factors are the same for both turbines [1-4]. Under these circumstances,

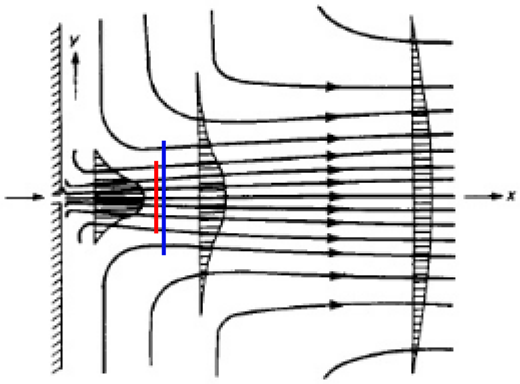
$$\frac{P_1}{P_2} \approx \left(\frac{R_1}{R_2}\right)^2. \tag{5}$$

Accordingly, if $R_1 > R_2$, $\frac{P_1}{P_2} > 1$. However, if the wind speed is not uniform but varies along the length of the turbine blade, then, the average wind speed over the length of the shorter blades will be different from that over the length of the longer blades. Equation (4) indicates that, when the variation of the interference factor with the length of the blade can be neglected, the turbine with the longer blades may not necessarily produce more power than the one with shorter blades. Indeed, from Equation (4), it can be seen that for each ratio of radii, $\frac{R_1}{R_2} > 1$, a corresponding ratio of speeds, $\frac{V_1}{V_2} < 1$, can always be found to satisfy the inequality $\frac{P_1}{P_2} < 1$. Whether or not this can actually happen in practice remains to be demonstrated. The purpose of this article is to present data that show that turbine T_2 can produce more power than turbine T_1, when both turbines are immersed in a wind stream that has a non-uniform distribution of speeds.

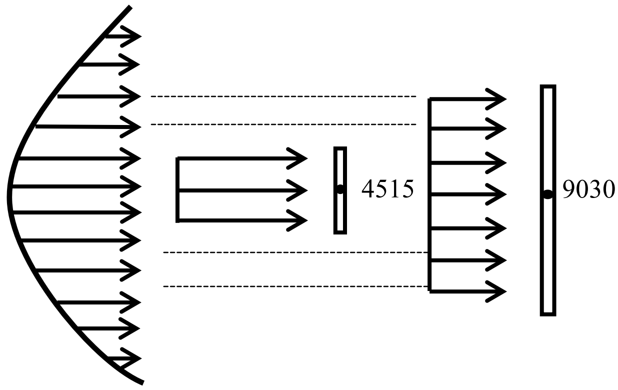
The article is organised in the following manner: first, the exact solution of the flow of a laminar free jet in the plane is presented as an example of flow conditions in which a turbine with shorter blades can produce more power than one with longer blades. This is followed by the presentation of the design of four miniature turbines that were used during testing. After that, the results from the flow from the box fan that was tested are used as an approximation for a flow in which the distribution of speeds is non-uniform. Next, results obtained from testing the performances of the four miniature turbines using a box fan are presented. Finally, these results are compared and interpreted to show the effects of one type of non-uniform flow on the power produced by miniature turbines.

FLOW OF A LAMINAR FREE JET IN THE PLANE

The magnitude of the wind speed can vary in different ways; for example, it can decrease monotonically away from the axis of the turbine; it can also increase monotonically away from that axis. An example of the former is flow of a laminar free jet in the plane [7]. An example of the latter is flow in the wake of a large body [8][9]. Analysis that uses flow of the laminar free jet suggests that under those flow conditions, the turbine with the shorter blade could generate more power than the one with the longer blade. How this can happen in practice has been illustrated graphically in Figure 1, where Figure 1a shows the side view of a hypothetical setup in which two turbines have been inserted in the air stream generated by a laminar free jet. If the turbines are placed symmetrically with respect to the axis of the jet, the average air speed on the shorter blades will be higher than that on the longer blades. The relative magnitudes of such speeds are illustrated in Figure 1b. Accordingly, when the wind speed is non-uniform along the length of the blades, a turbine with shorter blades can theoretically produce more power than one with longer blades.



a)



b)

Figure 1: a) A series of two-dimensional velocity profiles (plotted from Equation (6)) of a laminar free jet superimposed onto a pattern of streamlines for that flow [7][10]; b) A graphical representation of a non-uniform velocity distribution and the averaged air speeds on the faces of two turbine blades of different radii. Turbines 4515 and 9030 are shown here. These turbines are described below.

This graphical illustration can be supported analytically by using an equation that gives the distribution of the speed along the turbine blades in the case of flow from a laminar free jet. The velocity distribution in a laminar free jet that issues from a pinhole in a wall and into a plane was derived by Schlichting [7]. If the pinhole is the origin of an x-y coordinate system, as shown in Figure 1a, then, that distribution is given by Equation (6), shown below [10]:

$$u(x,y) = u_{max} sech^2 \left[0.2752 \left(\frac{J\rho}{\mu^2 x^2} \right)^{1/3} y \right], \tag{6}$$

where $u_{max} = 0.4543 \left(\frac{J^2}{\rho \mu x} \right)^{1/3}$ is the maximum speed, which occurs on the axis of symmetry of the jet, J is the momentum flux across any (x = constant) cross section and μ is the absolute viscosity of the fluid. The momentum flux, J, is presumed constant in the derivation. The mass rate of flow across any (x = constant) plane is given by [10]:

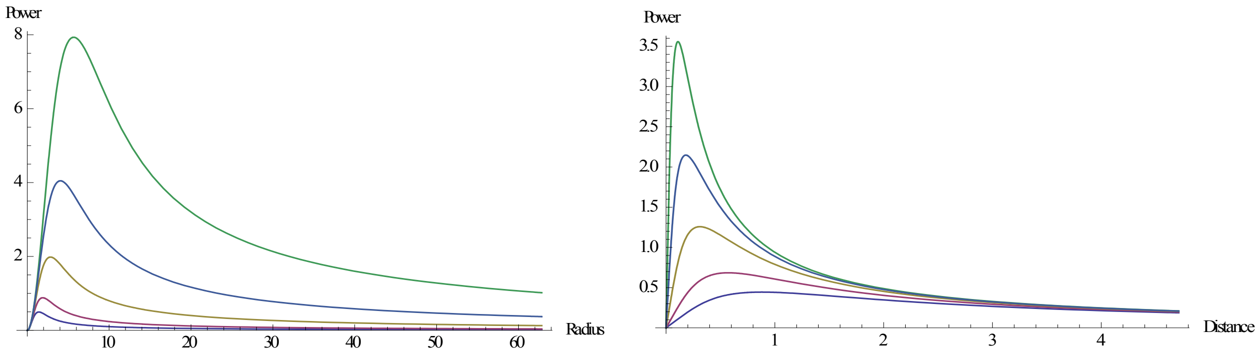
$$\dot{m} = \rho \int_{-\infty}^{+\infty} u dy = 3.302 (J\rho\mu x)^{1/3} \tag{7}$$

The average speed of the wind over a turbine blade immersed in a laminar free jet can be expressed as a fraction of the maximum speed that is measured on the axis of the jet. Thus, $\frac{u_{ave}}{u_{max}} = \alpha$, where:

$$\alpha = \frac{tanh\left[0.2752\left(\frac{J\rho}{\mu^2 x^2}\right)^{1/3} R\right]}{\left[0.2752\left(\frac{J\rho}{\mu^2 x^2}\right)^{1/3} R\right]} \tag{8}$$

Integrating this expression into Equation (1), the power produced by a turbine in a laminar free jet then becomes:

$$P = 0.5892\left(\frac{a(1-a)^2}{\mu}\right)\left(\frac{R^2 J^2 \alpha^3}{x}\right). \tag{9}$$



a) b)

Figure 2: A series of curves, plotted from Equation (9), showing how power produced by a turbine varies a) with the radius of the turbine blade, at fixed distances from the turbine; and b) with distance between the fan and the turbine, at fixed blade radii. In both plots, a fixed rate of mass flow is assumed to issue from the fan.

Figure 2 shows that at relatively small radii, power increases with increases in the radius of the turbine, just as in uniform flow. However, beyond a critical radius, power decreases as one increases the length of the turbine blade. In order to test whether or not these conditions can be created in the laboratory, one needs turbines with blades of different lengths and an air stream in which the speed distribution varies along the length of the turbine blades. To meet the former condition, four different turbines were designed in the laboratory. These turbines were identical in every respect except for the lengths of the turbine blades. To meet the latter condition, a box fan with three speeds was used as a source of wind. Experimental data collected in the laboratory showed that the distributions of speeds at many cross sections along the wind stream issuing from the fan were non-uniform and their non-uniformity was somewhat similar to that of a plane laminar free jet that is shown in Figure 1a.

DESIGN OF FOUR MINIATURE WIND TURBINES

Four miniature wind turbines were assembled using the same materials and tested using experimental setup as in previous experiments [1-3]. Three rectangular blades were used in each of the four turbines. Many angles and wind speeds were tested to determine the relationship between the size of the blades and the amount of power that each turbine was able to produce. In order to vary the wind speed, the fan was moved away from the wind turbine in predetermined increments in order to decrease the wind speed on the turbine blades progressively [1-3]. At each new position of the fan, the voltage and current generated by the turbine were recorded. Power produced by each turbine was computed using the voltage and current readings that were recorded. Angles were varied between 0˚ - 90˚ and the distances between 1ft and 21 ft. Each turbine was assigned a unique model number consisting of four digits developed to designate the dimensions of the different turbine blades. These model numbers are shown in Table 1 together with the dimensions of the corresponding turbine blades. That table also shows the aspect ratios of the blades, the areas that are swept by these blades during motion, as well as the solidity of the blades.

The aspect ratio a_r is the ratio between the length L and width w of each blade. The aspect ratio is calculated by using Equation (10). By design, all constructed blades had the same aspect ratio. The aspect ratio was held constant in these experiments in order to 1) make the blades have proportional dimensions; and 2) determine the effect of the size of the swept area on the amount of power produced when the aspect ratio is held constant:

$$a_r = \frac{L}{w} \tag{10}$$

The swept area A_s is the circular area that the leading edge of a turbine blade covers during one full rotation of the blade. The swept area is calculated by using Equation (11), where R denotes the radius of the blades:

$$A_s = \pi R^2 \tag{11}$$

It can be seen that the swept area decreases as the length of the blade decreases. Thus, longer blades will cover a larger portion of the velocity profile of the wind when compared to shorter blades. It follows that, when the speed distribution is not uniform, the average speed to which a swept area is exposed depends upon its size.

The solidity σ is a dimensionless number that represents the ratio of the total area occupied by all the blades that are attached to a given hub to the swept area. In Equation (12), the number of blades on the turbine n is multiplied by the width of the blades, the length of the blades, and divided by the swept area. The solidity was also held constant because the dimensions of the blades are all proportional to each other:

$$\sigma = \frac{nwL}{\pi R^2} \tag{12}$$

Table 1: The characteristics of the four turbine blades that were used in experiments.

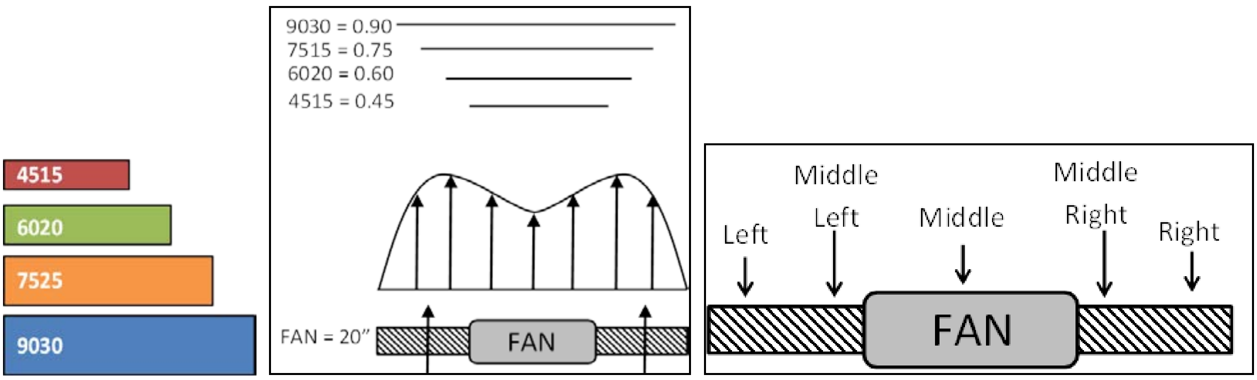
Model #	Dimensions (in)	Aspect Ratio	Swept Area (in^2)	Solidity
4515	4.5 x 1.5	3	63.62	0.318
6020	6.0 x 2.0	3	113.10	0.318
7525	7.5 x 2.5	3	176.71	0.318
9030	9.0 x 3.0	3	254.47	0.318

The blade dimensions that were outlined in Table 1 are depicted graphically in Figure 3, where Figure 3a compares the shapes and sizes of the four blades and Figure 3b the size of the diameter of each set of blades to that of the diameter of the fan. The diameter of the 9030 blades is 90% that of the fan, while the diameter of the 4515 blades is 45% that of the fan. Blades 6020 (60%) and 7525 (75%) were between these two. It can be seen that the 4515 blades could be immersed more deeply into the air stream from the fan than the larger 9030 blades, which were nearly the same size as the diameter of the fan.

NON-UNIFORM FLOW FROM A BOX FAN

It was determined experimentally that the distribution of the magnitude of wind speeds inside the air stream produced by the fan was not uniform. For each speed setting of the fan, profiles of the wind velocity were measured along many cross sections of the air stream produced by the fan. Figure 3b shows one such velocity profile. Profiles were obtained experimentally by sampling the speed of the air in the stream that issued from the fan at five different points along a given flow cross section, as shown in Figure 3c. The *Middle* position registered less wind than expected because it was located directly in front of the rotor of the fan.

The experimental setup and procedure were the same as those described in previous articles [1-3]. The four turbines were tested under the same conditions in order to compare their performances. Twelve series of tests were conducted. A complete series consisted of testing one turbine under the following conditions: at a fixed speed setting, a specific wind speed was created by placing the fan successively at 21 different distances between the turbine and the fan. This was repeated using all four turbines and all three fan speeds. Figure 5, in the appendix, shows a sketch of the experimental set up.

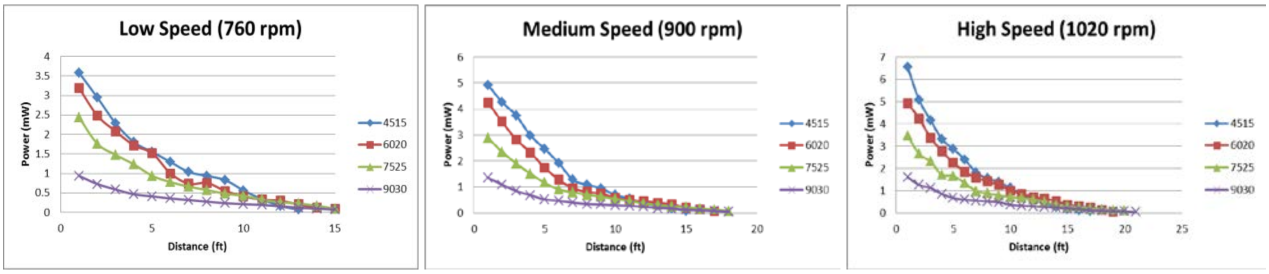


a) b) c)

Figure 3: a) A proportional representation of the size of each set of turbine blades; b) A velocity profile of the flow from a fan and the ratios of the diameters of the four kinds of turbine blades tested to the diameter of the fan that was used during experiments; and c) The five locations in the cross section of the flow from the fan where the air speed was sampled using an anemometer. Sampling was done at many flow cross sections.

The results from these series of tests are shown for different fan speeds in Figures 4a, 4b and 4c, respectively. They show data collected using angles of inclination of turbine blades at which the turbines produced their maximum power [1-3]. Results show that 1) as expected, the magnitude of the speed of the wind affects the amount of power produced by the turbine; since the closer the fan was to the turbine, the higher the speed of the wind that crossed the turbine blades, these plots show that power increased with wind speed; and 2) turbines with shorter blades produced more power than those with longer blades.

This was the case regardless of whether the fan speed was low, medium or high. Specifically, these plots show that the 4515 turbine produced the most power, followed, respectively, by turbines 6020, 7515 and 9030, in that order. These performances are in reverse order of what one would expect when the turbines are subjected to wind speeds that are uniform. Furthermore, the plots shown in Figure 4 are in qualitative agreement with those found on the down-sloping side of Figure 2b, which were obtained from the analysis of the laminar free jet.



a) b) c)

Figure 4: Power produced by four different turbines versus distance between the fan and the turbine under three different conditions: a) low fan speed (760 rpm); b) medium fan speed (900 rpm); and c) high fan speed (1020 rpm).

CONCLUSIONS

The distribution of wind speeds in the airflow from a box fan shows that the distribution is non-uniform in the following way: the magnitudes of the wind speeds decrease away from the centre of the fan. Therefore, for turbines that are placed symmetrically with respect to the axis of the fan, the average wind speeds on the shorter blades are higher than those on the longer blades. Data on the power produced by four different miniature wind turbines subjected to such non-uniform wind speeds were collected and analysed. Results show that such non-uniform distributions of wind speeds lead to turbine performances that are contrary to what one would expect when wind speeds are uniform.

Specifically, power produced by the tested wind turbines decreased as the lengths of the turbine blades increased. When the fan was close to the turbine, the wind speeds on the blades and their gradients increased appreciably, accentuating the differences in the average speeds and, hence, in the power produced by turbines equipped with blades of different lengths. Airflow from the fan becomes closer and closer to uniform further and further away from the fan. Unfortunately, at such distances, the magnitudes of the speeds of air become small, making such distances impractical to use, because not enough power is produced by turbines under such conditions. The use of fans to test turbines must account for the non-uniform flows produced by them.

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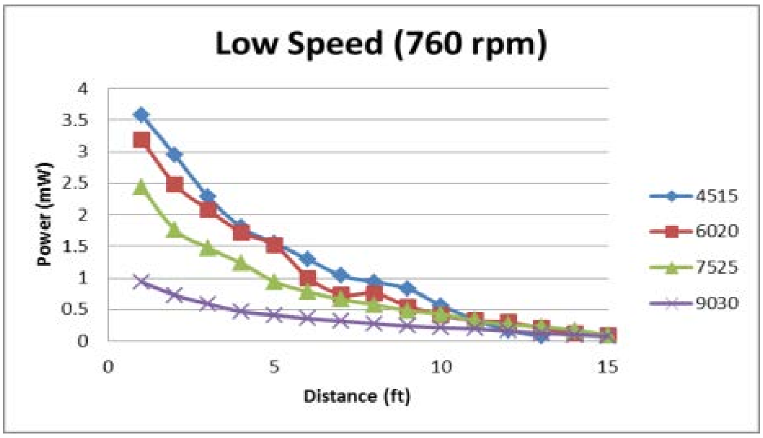
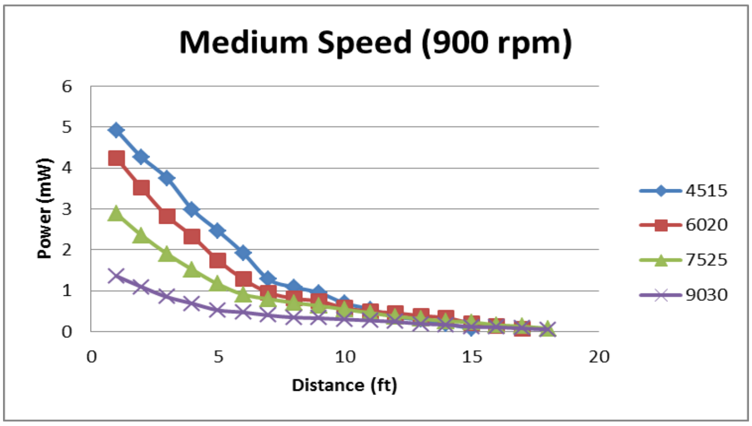
APPENDIX

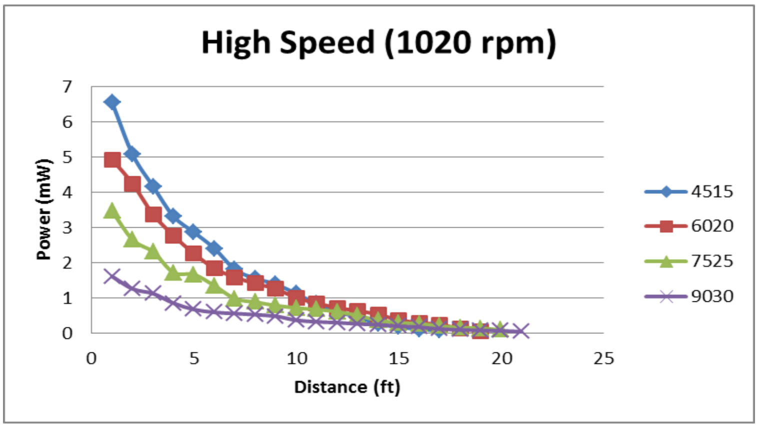
Experimental data gathered using the four turbines at each of the three speeds of the box fan are shown in Figure 5 below as an enlargement of Figure 4.



a) Data collected with the fan running at low speed (760 rpm).



b) Data collected with the fan running at medium speed (900 rpm).



c) Data collected with the fan running at high speed (1020 rpm).

Figure 5: Enlarged plots of power produced by four different turbines versus distance between the fan and the turbine, for three fan speeds: a) low fan speed (760 rpm); b) medium fan speed (900 rpm); and c) high fan speed (1020 rpm).